KNT/KW/16/5169

Bachelor of Science (B.Sc.) Semester—V (C.B.S.) Examination

MATHEMATICS

Paper—2

(M_{10}-Metric Space, Complex Integration and Algebra)

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all the ***five*** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) Let $\{E_n\}$, n = 1, 2, 3 ..... be a sequence of countable sets, then prove that $S = \bigcup_{n=1}^{\infty} E_n$ is countable. 6

(B) Show that the function d defined by $d(x, y) = | x - y | \ \forall \ x, y \in R$, is a metric on the set R of real numbers. 6

OR

(C) Prove that every neighbourhood is an open set. 6

(D) For any collection $\{F_\alpha\}$ of closed sets prove that $\bigcap_{\alpha} F_\alpha$ is closed. Give an example to show that arbitrary union of closed sets is not closed. 6

UNIT—II

2. (A) Prove that closed subsets of compact sets are compact. 6

(B) If $\{I_n\}$ is a sequence of intervals in R^1 such that $I_n \supset I_{n+1}$ (n = 1, 2, 3....), then prove that $\bigcap_{n=1}^{\infty} I_n$ is not empty. 6

OR

(C) Prove that every convergent sequence in a metric space is a Cauchy sequence. 6

(D) Prove that every bounded infinite subset R^k has a limit point in R^k. 6

UNIT—III

3. (A) Show that a commutative ring D is an integral domain if and only if for a, b, c $\in$ D with $a \neq 0$, the relation $ab = ac$ implies $b = c$. 6

(B) If R is a ring and L is a left ideal of R.
Let $\lambda(L) = \{x \in R / xa = 0 \text{ for all } a \in L\}$. Prove that $\lambda(L)$ is a two-sided ideal of R. 6

OR

(C) If ϕ is a homomorphism of R into R′ with Kernel $I(\phi)$ then prove that :
(i) $I(\phi)$ is a subgroup of R under addition.
(ii) If $a \in I(\phi)$ and $r \in R$ then both ar, ra $\in I(\phi)$. 6

(D) If U is an ideal of a ring R, then prove that R/U is homomorphic to R. 6

UNIT—IV

4. (A) Evaluate $\oint_C \frac{4-3z}{z(z-1)(z-2)} dz$ using Cauchy's integral formula where C is the circle $|z| = \frac{3}{2}$. 6

(B) Verify Cauchy's theorem for $\int_C z^3 dz$ over the boundary of the triangle with vertices (1, 2), (1, 4), (3, 2). 6

OR

(C) State and prove Cauchy's Residue theorem for an analytic function. 6

(D) Evaluate the residue of $\frac{z^2}{(z-1)(z-2)(z-3)}$ at 1, 2, 3 and infinity and show that their sum is zero. 6

Question—5

5. (A) Prove that $\left(\bigcup_\alpha G_\alpha\right)^C = \bigcap_\alpha G_\alpha^C$. 1½

(B) Define atmost countable and uncountable sets. 1½

(C) If A = [0, 1] an B = (1, 2), then show that A and B are not separated in R. 1½

(D) Define K-cell and explain 2-cell. 1½

(E) Let R be a ring and a, b $\in$ R. Prove that $(a + b)^2 = a^2 + ab + ba + b^2$. 1½

(F) If ϕ is a homomorphism of ring R into R′, then show that $\phi(0) = 0'$ where $0' \in R'$. 1½

(G) Evaluate $\int_C \frac{1}{z^2-1} dz$, where C is the circle $x^2 + y^2 = 4$. 1½

(H) Evaluate $\int_0^{1+i} (x - y + ix^2) dz$ along the straight line from z = 0 to z = 1 + i. 1½